FLOW AROUND AXISYMMETRIC BODIES WITH THREE CONSTANT-VELOCITY SECTIONS

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Of interest is the determination of the shape of thick axisymmetric bodies having a small surface area for a given volume, and simultaneously, a sufficiently high value of the critical Mach number for a small pressure drop on the body surface which would permit realization of unseparated flow.

As is known [1], for given length and volume of an axisymmetric body the maximal value of the critical Mach number is achieved when the body consists of two disks connected by a surface on which the velocity is everywhere sonic. An analog of this flow in an incompressible fluid is the Ryabouchinsky flow for a disk. However, it is extremely difficult to assure unseparated flow around such a body because of the high value of the pressure drop on its surface.

As Chaplygin [2] has shown for the plane flow case, any stagnation point on a body surface be replaced by a finite domain with fluid at rest adjacent to the body. Flows of this kind for a cylinder and sphere were first obtained in [3]. As all flows with free boundaries, such flows possess a number of extremal properties. Let us note some of them. Let the potential flow around a plane body that is symmetric relative to the x axis, or an axisymmetric body with axis of symmetry x, be considered. A homogeneous stream, directed along this axis, exists at infinity. Let the curve L be a fixed part of the body contour in the upper half-plane, and L_* the variable part of the body contour connecting the point M_* on the axis of symmetry x and the point M on the contour L. In the axisymmetric case L and L_* are sections of the fixed and variable body surface in the meridian plane. It is shown in [4] that for a given location of the point M sufficiently close to the axis of symmetry, the maximal value of the square of the velocity on the variable part of the body L_* reaches a minimum if and only if the velocity on L_* is constant everywhere, i.e., when L_* is a free streamline. The proof of this result is based on utilization of the maximum principle for the stream functions of plane and axisymmetric flows. It can analogously be shown that such flows have still another extremal property: For a given location of the point M_* on the axis of symmetry x, the minimal value of the square of the velocity on the variable part of the body L_* reaches a maximum if and only if the curve L_* is a free streamline. The magnitude of the velocity on L_* is related uniquely to the position of the point M_*.

Therefore, by applying Ryabouchinsky flow at the stagnation points of a domain with constant pressure, we achieve a maximal diminution in the pressure drop on the surface for a fixed increase in the length of the body obtained by such means. The idea of constructing such bodies is due to Taganov. A family of flows of such type is obtained in this paper for the incompressible fluid case. To diminish the pressure gradients on the body surface, spheres are used as reference bodies instead of disks. For given body length and volume this results in a certain increase in the maximal value of the perturbed flow velocity. Taking account of compressibility of the fluid will result only in a quantitative refinement of the results obtained in this paper.

Let us consider the axisymmetric flow around a body of shape unknown in advance, which is symmetric relative to the plane x = 0 in an x, r coordinate system (Fig. 1). On the forward section of the body AB the velocity is constant and equal to v_1, the section BC is a part of the surface of a sphere, and the velocity is also constant, and equal to v_2, on the middle part CC' of the body. A potential stream homogeneous at infinity, and directed along the x axis, flows around the body. The flow velocity at infinity and the radii of the sphere are taken equal to one. In addition to the velocities v_1 and v_2, the problem contains the following parameters: l_1 is the distance between the body nose and the center of the nearest sphere, l_2 is the distance between the centers of the spheres, and α_1 and α_2 are angular

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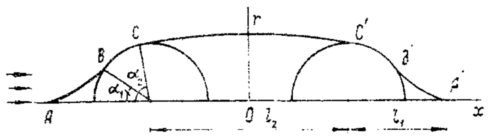

Fig. 1

locations of the points B and C. We determine the location of the point C (the parameter α_2) from the condition that the velocity gradient vanish upon approaching the point C along the sphere surface. It is known from the theory of an ideal fluid jet [5] that this condition is equivalent to the condition of "smooth separation" of Brillouin—Will: The curvature of a surface with a constant value of the velocity agrees with the curvature of the body at the separation point. Analysis of the known solutions for jet flows shows that the problem should have a two-parameter family of solutions. The results of a numerical solution confirm this assumption.

The problem is formulated as follows: For given values of the parameters l_1 and l_2 determine the parameters v_1, v_2, α_1, α_2, the shape of the surfaces AB and CC' and the Stokes stream function ψ, satisfying the differential equation $\psi_{xx} + \psi_{rr} - \psi_r/r = 0$, resulting from the condition that the flow is potential, and the following boundary conditions: $\psi = 0$ on the whole body surface, $|\nabla\psi|/r = v_1$ on AB and A'B', $|\nabla\psi|/r = v_2$ on CC', and $|\nabla\psi|/r \to r^2/2$ as $x^2 + r^2 \to \infty$.

Considering the body surface as a vortical surface with unknown intensity, the stream function can be represented in the form [6]

$$\psi(\mathbf{r}) = \frac{r^2}{2} + \frac{1}{4\pi}\int\limits_L \gamma(\mathbf{r}')\, Q(\mathbf{r}, \mathbf{r}')\, dl, \tag{1}$$

where

$$Q(\mathbf{r}, \mathbf{r}') = p[(2 - k^2)K - 2E]; \; p = [(x - x')^2 + (r + r')^2]^{1/2};$$

$k = (4rr')^{1/2}/p$ is the modulus of the complete elliptic integrals of the first and second kinds K and E, $\mathbf{r} = (x, r)$, $\mathbf{r}' = (x', r')$ are points on the body surface with respect to which the integration is performed, L is the curve ABCC'B'A', l is the length of an arc of the curve L, γ is the intensity of the vortex layer on the body surface. On the surfaces AB and A'B' we have $\gamma = -v_1$, while $\gamma = -v_2$ on CC', and $\gamma = \omega(l)$ on the streamlined surface of the sphere BC and B'C'.

The function ω should satisfy the following boundary conditions [6]:

$$\omega(B) = -v_1, \; \omega(C) = -v_2, \; \omega_l\,(C) = 0. \tag{2}$$

The magnitude of the stream velocity at the point A is v_1; hence, the condition [7]

$$v_1 = 1 + \frac{1}{2}\int\limits_{-a}^{a} \gamma(x)\,\frac{r^2(1 + r_x^2)^{1/2}}{[r^2 + (x + a)^2]^{3/2}}\,dx, \tag{3}$$

should be satisfied at this point, where $a = l_1 + l_2/2$, and (x, r) is a point on the body surface.

From the condition that the stream function (1) vanish on the body surface, we obtain an integral equation to determine the function ω, the unknown body surface shape, and the parameters v_1, v_2, and α_1. Conditions (2) and (3) should be satisfied here. The parameter α_2 is determined from the "smooth separation" condition.

The method for the numerical solution of the problem is a development of the method used in [6] to compute Ryabouchinsky flow.

Taking account of the infinite velocity gradient on the sphere surface at the point B and the last condition in (2), we represent the intensity of the vortex layer on the section BC in the form

$$\omega(l) = c_1(l^{1/2} - l/2 - 1/2) + \sum_{n=2}^{N} c_n \cos\pi(n - 2)\,l, \quad 0 \leqslant l \leqslant 1.$$

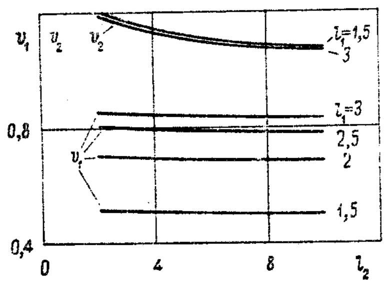

Fig. 2

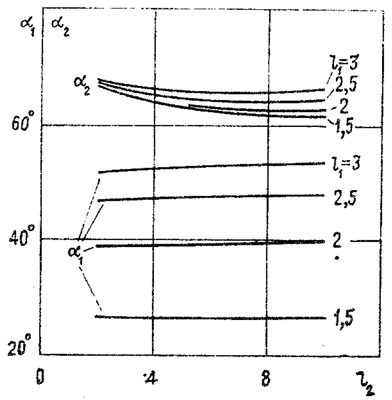

Fig. 3

For a given body shape and given values of the parameters l_1, l_2, and α_2, we obtain a system of linear equations in the parameters v_1, v_2 and c_i ($i=1, \ldots, N$) from the condition that the stream function (1) vanish at N points on the streamlined part of the sphere surface, the first two conditions in (2), and the relationship (3). The parameter α_1 is nonlinear in these equations. After having solved the system of equations obtained, iteration is performed for the shape of the boundary on which the velocity should be constant. The shape of the body surface on the section CC' was given in the form of a cubic spline in the variable x. To take account of the infinite curvature of the body contour at the points A and B, the radius of the section of the initial portion of the body AB was given by a cubic spline in the argument t, and the coordinate x was determined by the relationship

$$x(t) = (1 - \cos \pi t)(l_1 - \cos \alpha_1)/2 - l_1 - l_2/2,\ 0 \leqslant t \leqslant 1.$$

Iterations for the body radius at the spline nodes were performed as follows:

$$r^{k+1} = r^k - \frac{\psi(r^k)}{r^k v_e}\left(1 + r_x^{k^2}\right)^{1/2}.$$

Here r^k is the radius of the body section at the spline node for the preceding approximation, r^{k+1} is the succeeding approximation for r^k, $v_e = v_1$ on the initial section of the body AB, $v_e = v_2$ on the middle section CC'. The convergence of this iteration process turned out to be considerably better than that applied in [6].

After the new approximation had been obtained for the unknown body surface shape, the following iteration was performed: Again the condition of nonpenetration was satisfied on the streamlined surface of the sphere and conditions (2) and (3) were satisfied, and then the next approximation for the body shape was determined.

Determination of the initial location of the beginning of the middle part of the body surface with the constant velocity was performed by the method of [6].

The computations were executed for values of the parameters $1.5 \leqslant l_1 \leqslant 3$ and $2 \leqslant l_2 \leqslant 10$. The magnitudes of the remaining parameters are represented in Figs. 2 and 3. As is seen, in this range of variation of l_1 and l_2 the parameters characterizing the middle section of the body with the constant velocity CC' (see Fig. 1) and the sections AB and A'B', depend weakly on each other. For an unbounded increase in the length of the forward and rear section with constant velocity, evidently $\alpha_1 \to 90°$, $\alpha_2 \to 90°$, $v_1 \to 1$, $v_2 \to 1$ and the flow is the unperturbed flow around a cylinder with radius one. As $l_1 \to 1$ we arrive at the flow around a sphere by the Ryabouchinsky scheme obtained in [6]. The results in this paper agree well with the results of [6] for values of the parameter v_2. Somewhat worse is the correspondence for the angular location of the beginning of the middle section with constant velocity. This is apparently explained by the less successful selection in this paper for the approximation of the free streamline surface contour on the middle section of the body in the neighborhood of the point of jet descent.

Meridian sections of the three bodies obtained and the stream velocity distribution along their surfaces are presented in Fig. 4. The forward and rear sections of the body have infinite curvature at juncture points with the spherical surfaces. The velocity gradient on the spheres becomes infinite at these points. We have a second order tangency and

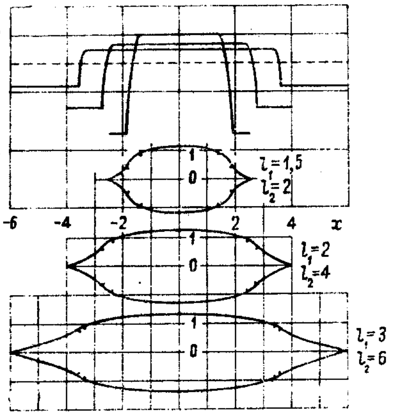

Fig. 4

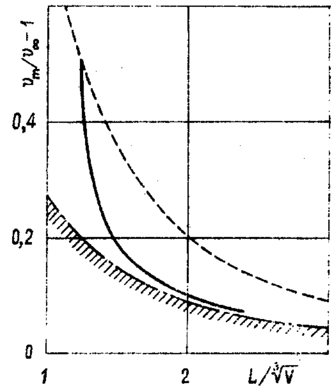

Fig. 5

zero velocity gradient for junctures of the middle part of the body with the surfaces of the spheres.

The following notation is used in Fig. 5: v_m is the maximal value of the perturbed stream velocity, v_∞ is the magnitude of the unperturbed velocity, L is the body length along the axis of symmetry, and V is the body volume. The shaded line corresponds to ellipsoids of revolution, and the solid line to the family of bodies obtained in this paper without taking account of the volume and length of the pointed domains with constant pressure appended to the spheres. The dash-dot line is the lower bound of the maximal values of the perturbed flow velocity. It corresponds to the flow around disks according to the Ryabouchinsky scheme and is constructed from the results in [6]. The bodies obtained in this paper for distances between the centers of the reference spheres that exceed their diameter have maximal values of the perturbed flow velocity that are close to the minimal possible and almost half that for ellipsoids of revolution.

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